Spin current through a tunnel junction

Matthias Braun[1], Jürgen König[1], Jan Martinek[2,3]

[1] *Institut für Theoretische Physik III, Ruhr-Universität Bochum, 44780 Bochum, Germany*

[2] *Institute of Molecular Physics, Polish Academy of Science, 60-179 Poznań, Poland*

[3] *Institut für Theoretische Festkörperphysik, Universität Karlsruhe, 76128 Karlsruhe, Germany*

Abstract

We derive an expression for the spin-current through a tunnel barrier in terms of many-body Green's functions. The spin current has two contributions. One can be associated with angular-momentum transfer by spin-polarized charge currents crossing the junction. If there are magnetic moments on both sides of the tunnel junction, due to spin accumulation or ferromagnetic ordering, then there is a second contribution related to the exchange coupling between the moments.

Key words:
PACS: 72.25.Mk, 72.25.Pn, 72.25.-b, 73.23.Hk, 85.75.-d

1 Introduction

In recent years spintronics and magneto-electronic devices have been a major research topic, biased in part by the prospect of new technological applications in information processing based on the simultaneous use both the electron's spin and charge degree of freedom [1]. All spintronic devices depend on driven spin currents between different subsystems, which is why a deep understanding of these spin-currents is essential for this field of research.

A naive identification of total transfer of angular momentum with the spin-polarized charge current passed through a junction, $I_{\text{spin}} = I_\uparrow - I_\downarrow$, neglects the possibility of spin transfer due to exchange couplings. The aim of this paper is to derive an expression for the spin current through a tunnel junction, that is suitable to describe interacting electron systems, such as quantum dots,

Preprint submitted to Elsevier Science *2 February 2008*

and that explicitly demonstrates the two different contributions to the spin current.

2 Derivation of the spin current through a tunnel barrier

We start our discussion with the calculation of the spin current between a ferromagnetic lead L and an island I via a tunnel contact. In this first section, we do not specify the electronic structure of the island yet, i.e., any kind of many-body effects or couplings of the island to other leads are included. The Hamiltonian of such a tunnel system is given by

$$H = \sum_{k\alpha} \varepsilon_{k\alpha} c^\dagger_{k\alpha} c_{k\alpha} + H_{\mathrm{int}}(\{d^\dagger_{p\gamma}\}; \{d_{p\gamma}\}) + \sum_{k\alpha,p\gamma} \left(V_{k\alpha,p\gamma} c^\dagger_{k\alpha} d_{p\gamma} + h.c. \right) , \quad (1)$$

where $c^\dagger_{k,\alpha}$ are the fermion creation operators in the lead and $d^\dagger_{p,\gamma}$ are the corresponding operator for the island. In the lead (island) we label the momentum states of the electrons with k (p) and the spin with α and β (γ and δ).

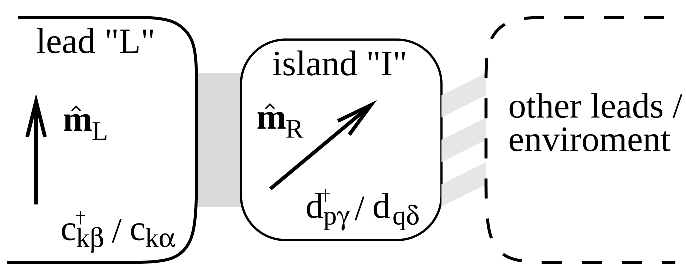

Fig. 1. A tunnel junction connecting a ferromagnetic lead to an island.

Starting from this Hamiltonian we calculate the spin current in close analogy to the derivation of the charge current by Meir and Wingreen for interacting electron systems [2]. If the spin is a conserved quantity, the time derivative of the total spin in the lead equals the spin current through the tunnel barrier $\boldsymbol{J}_{\mathrm{L}} =< \dot{\boldsymbol{S}}_{\mathrm{L}} >$. In the Heisenberg picture, the time evolution of the spin operator $\boldsymbol{S}_{\mathrm{L}} = (\hbar/2) \sum_{k\alpha\beta} c^\dagger_{k\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{k\beta}$ is given by $i\hbar \dot{\boldsymbol{S}}_{\mathrm{L}} = [\boldsymbol{S}_{\mathrm{L}}, H]$, which yields

$$\begin{aligned} \boldsymbol{J}_{\mathrm{L}} &= \frac{-i}{2} \sum_{k,p} \left(V_{k\alpha,p\gamma} \boldsymbol{\sigma}^\star_{\alpha\beta} \langle c^\dagger_{k\beta}\, d_{p\gamma} \rangle - V^\star_{k\alpha,p\gamma} \boldsymbol{\sigma}_{\alpha\beta} \langle c_{k\beta}\, d^\dagger_{p\gamma} \rangle \right) \qquad (2) \\ &= \frac{-1}{2} \sum_{k,p} \int \frac{d\omega}{2\pi} \left(V_{k\alpha,p\gamma} \boldsymbol{\sigma}^\star_{\alpha\beta}\, G^<_{p\gamma,k\beta}(\omega) - V^\star_{k\alpha,p\gamma} \boldsymbol{\sigma}_{\alpha\beta}\, G^<_{k\beta,p\gamma}(\omega) \right) , \end{aligned}$$

where we introduced the Keldysh Green's functions $G^<_{p\gamma,k\beta}(t) = i\langle c^\dagger_{k\beta}(0)\, d_{p\gamma}(t)\rangle$. By use of a Dyson equation [2] we can replace the latter with (free) Green's functions $g_{k\beta}(\omega)$ of the lead and Green's functions $G^<_{q\delta,p\gamma}(t) = i\langle d^\dagger_{p\gamma}\, d_{q\delta}(t)\rangle$ of the island. By choosing the magnetization direction $\hat{\mathbf{m}}_{\mathrm{L}}$ as spin-quantization axis, the lead Green's functions $g_{k\sigma}$ are diagonal in the spin index. The lead Green's functions are then $g^<_{k\sigma} = 2\pi i f^+_{\mathrm{L}}(\omega)\delta(\omega - \varepsilon_{k\sigma})$, $g^>_{k\sigma} = -2\pi i f^-_{\mathrm{L}}(\omega)\delta(\omega - \varepsilon_{k\sigma})$, $g^{\mathrm{ret}}_{k\sigma} = 1/(\omega - \varepsilon_{k\sigma} + i0^+)$, and $g^{\mathrm{adv}}_{k\sigma} = (g^{ret}_{k\sigma})^\star$. There f^+_{L} stands for the Fermi distribution function in the lead L and $f^-_{\mathrm{L}} = 1 - f^+_{\mathrm{L}}$.

Assuming, furthermore, that tunnel events conserve the spin of the electrons, we substitute the tunnel matrix elements by $V_{k\alpha,p\gamma} = t_{k,p} \cdot \delta_{\alpha\gamma}$, and define the spin-dependent transition rates $\Gamma^\gamma_{q,p} = \sum_k t_{k,q} t^\star_{k,p} \delta(\omega - \varepsilon_{k\gamma})$. After a lengthy but straightforward calculation, the spin current can be written as

$$\begin{aligned}\boldsymbol{J}_{\mathrm{L}} = \frac{-2\pi i}{4} \sum_{p,q} \int \frac{d\omega}{2\pi} \quad & \boldsymbol{\sigma}_{\gamma\delta}(\Gamma^\gamma_{q,p} + \Gamma^\delta_{q,p}) \left[f^+_{\mathrm{L}}(\omega)\, G^>_{q\delta,p\gamma} + f^-_{\mathrm{L}}(\omega)\, G^<_{q\delta,p\gamma} \right] \\ & + \boldsymbol{\sigma}_{\gamma\delta}(\Gamma^\gamma_{q,p} - \Gamma^\delta_{q,p}) \left[f^+_{\mathrm{L}}(\omega)\, (G^{\mathrm{ret}}_{q\delta,p\gamma} + G^{\mathrm{adv}}_{q\delta,p\gamma}) + \frac{1}{i\pi} \int' dE\, \frac{G^<_{q\delta,p\gamma}(E)}{E - \omega} \right]. \end{aligned} \tag{3}$$

This is the central most general result of our calculation. Since we did not specify the Green's functions $G_{q\delta,p\gamma}$ of the island yet, the expression for the spin current can be used for many situation, including strongly-correlated systems such as quantum dots [3].

By comparison with the expression of the charge current derived by Meir and Wingreen [2] for nonmagnetic systems, we identify the first line of Eq. (3) with a spin-polarized charge current. The origin of the spin-current contribution in the second line of Eq. (3) is the exchange interaction between lead and island. If the island does posses a magnetic moment, due to spin accumulation or ferromagnetic order, this moment couples to the magnetization of the lead and both precess around each other [4]. This exchange coupling changes the average spin on each side of the tunnel junction, and therefore it must also appear as contribution to the spin current crossing the tunnel barrier. In the latter case, the transfered angular moment is perpendicular to the magnetic moments of lead and island, which is sometimes described by "spin mixing conductances" [5].

3 Application to a FM-FM junction

For concreteness, we restrict the following discussion to the special case, that the island is an itinerant ferromagnet. Thereby the direction of magnetization $\hat{m}_{\mathrm{I}}$ of the island encloses a finite angle ϕ with the lead magnetization direction.

We further assume, that the island is large, to be also described as a reservoir in thermal equilibrium. Due to the non-collinear magnetization directions, the Green's function of the island is non-diagonal in the spin space $\check{G}_{p,p} = \check{U}(\phi)\,\mathrm{diag}(g_{p\uparrow}, g_{p\downarrow})\,\check{U}^{-1}(\phi)$ with the SU(2) rotations $\check{U}(\phi)$.

To simplify the result further, we assume that the absolute value of the tunnel matrix elements $|t_{k,p}| = |t|$ is independent of the momentum index. Then we can replace the transition rates $\Gamma^{\gamma}_{q,p} = \sum_k t_{k,q} t^{\star}_{k,p} \delta(\omega - \varepsilon_{k,\gamma})$ by the spin-resolved density of states $\rho_{\mathrm{L},\alpha} = \sum_k \delta(\omega - \varepsilon_{k,\alpha})$ and $\rho_{\mathrm{I},\gamma} = \sum_p \delta(\omega - \varepsilon_{p,\gamma})$. After performing all spin summations, we get in lowest-order in the tunnel coupling

$$\boldsymbol{J}_{\mathrm{L}} = \frac{\pi}{2} \int d\omega\, |t|^2 \Big(\Lambda_1(\omega) \hat{\mathbf{m}}_{\mathrm{L}} + \Lambda_2(\omega) \hat{\boldsymbol{m}}_{\mathrm{I}} + \Lambda_3(\omega) \hat{\mathbf{m}}_{\mathrm{L}} \times \hat{\mathbf{m}}_{\mathrm{I}} \Big) \tag{4}$$

$$\begin{aligned} \text{with} \quad \Lambda_1(\omega) &= [f^-_{\mathrm{L}}(\omega) f^+_{\mathrm{I}}(\omega) - f^+_{\mathrm{L}}(\omega) f^-_{\mathrm{I}}(\omega)] \chi_{\mathrm{L}}(\omega) \rho_{\mathrm{I}}(\omega) \\ \Lambda_2(\omega) &= [f^-_{\mathrm{L}}(\omega) f^+_{\mathrm{I}}(\omega) - f^+_{\mathrm{L}}(\omega) f^-_{\mathrm{I}}(\omega)] \rho_{\mathrm{L}}(\omega) \chi_{\mathrm{I}}(\omega) \\ \Lambda_3(\omega) &= \frac{1}{\pi} \int' dE\, \frac{f^+_{\mathrm{L}}(\omega) - f^+_{\mathrm{I}}(E)}{\omega - E} \chi_{\mathrm{I}}(E) \chi_{\mathrm{L}}(\omega) \end{aligned} \tag{5}$$

with the full density of states $\rho_{\mathrm{I}} = \rho_{\mathrm{I}\uparrow} + \rho_{\mathrm{I}\downarrow}$ and the spin-polarization density $\chi_{\mathrm{I}} = \rho_{\mathrm{I}\uparrow} - \rho_{\mathrm{I}\downarrow}$, and analogue definitions for the lead L.

In the first and second term we can recognize the spin current contribution of the charge transfer between the two reservoirs. With the cross product $\hat{\mathbf{m}}_{\mathrm{I}} \times \hat{\mathbf{m}}_{\mathrm{L}}$ the third term shows the typical structure for a precession movement. In the approach of spin dependent circuit theory [5] this spin current contribution corresponds to the imaginary part of the spin-mixing conductance.

4 Effect of exchange-coupling contribution

A very pronounced effect of a spin current is current-induced magnetization reversal [6]. However, there it is difficult to selectively address the spin-current contribution arising from the exchange interaction. The latter goal can be achieved, e.g., in measuring the charge current through a single-level quantum dot connected to two ferromagnetic leads. A current forced through such a quantum-dot spin valve will accumulate a non-equilibrium spin on the dot. This spin is sensitive to the exchange field generated by the leads. Its precession is predicted to be visible in the magneto-resistance of the device [3].

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